Math 201
Quiz #1, March 10, 2008

1. (20 pts.) Evaluate each of the following limits

$$(a)\lim_{n\to\infty}\left(\frac{4n-3}{4n+3}\right)^{n+1},(b)\lim_{n\to\infty}\frac{n^{\frac{1}{n}}\sin(3+n^5)}{\sqrt{n}},(c)\lim_{n\to\infty}\frac{\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{n^{24}}}{\ln n}.$$

2. (20 pts.) Decide the convergence or divergence of each of the following series:

$$(a)\sum_{n=1}^{\infty}\frac{3+\cos(n^{100})}{n\sqrt{n}},(b)\sum_{n=1}^{\infty}\left[\frac{1}{n^{1/3}}-\sin\left(\frac{1}{n^{1/3}}\right)\right],(c)\sum_{n=1}^{\infty}(\ln n)\sin\left(\frac{1}{n^{1.3}}\right).$$

3. (20 pts.) Find s_n the n^{th} **partial** sum of the series

$$\sum_{k=1}^{\infty}\left\{\frac{1}{\sqrt{2k-1}}-\frac{1}{\sqrt{2k+1}}\right\},$$

and then find the **sum** s of the series.

4. (20 pts.) Consider the series

$$\sum_{n=1}^{\infty}\frac{\ln n}{n}\left(\frac{2x-3}{4}\right)^n.$$

Find all values of x for which the series is convergent, distinguishing between absolute and conditional convergence.

5. (20 pts.) Let $f(x)=\ln(x^2+5x+4)$. Find the Taylor series expansion of f about the point $a=1$, and then, use this series, to find $f^{(n)}(1)$ for $n=0,1,2,3,\ldots$

Caution: The required expansion is about $a=1$.